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CONVECTIVE MASS TRANSFER IN A PERIODIC ARRAY OF SPHERES

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In problems of convective diffusion in a system of reacting particles at high Peclet numbers, the structure of singular streamlines which begin and end on particle surfaces plays an important role [1-3]. The flow involves chains of particles in which mass transfer is greatly retarded by the interaction of diffusion wakes and boundary layers of particles belonging to the chains. Taking account of the interaction of diffusion wakes and boundary layers of particles, and assuming that the ratio of the lattice period b to the radius a of a sphere satisfies the inequality $b/a \gg \mathrm{Pe}^{1/3}$, where Pe is the Peclet number of a single sphere, Voskanyan et al. [4] performed calculations for a system of spheres of equal radii at the nodes of a widely spaced cubic lattice. Under these assumptions the original problem could be reduced to a self-similar problem of the diffusion of matter with a constant concentration flowing past an isolated sphere [5]. In the present paper we consider mass transfer of a concentrated ordered system of reacting solid spheres when $b/a \ll \mathrm{Pe}^{1/3}$.

We consider steady convective diffusion in the laminar flow of a viscous incompressible liquid filtering through a system of reacting spheres of equal radii at the nodes of a cubic lattice. We assume that the liquid filters through the spaces between the spheres with an average velocity U which is parallel to one axis of the lattice, and that the Reynolds number $\mathrm{Re} = aU/\nu$, where ν is the kinematic viscosity of the liquid, is small. Then the velocity field of the liquid in the lattice can be determined within the framework of the cell model [6, 7], or when $b/a \gg 1$, by the concentrated-force model [4, 8]. Henceforth we assume that the position of a fixed sphere in the lattice is given by a set of three integers, and the distance along the stream axis is given by the value of the parameter $k = 1, 2, \ldots$.

Using a system of spherical coordinates with its origin at the center of an arbitrary sphere, the stream function near the surface of a sphere can be written in the form

$$\psi = (3/4)UA(n)(r-a)^2 \sin^2\theta, \quad \lim_{n\to 0} A(n) = 1,$$

where n is the number of spheres per unit volume. The specific expression for A(n) can be determined, in particular, from [4, 6-8].

The concentration distribution in the flow is determined by solving the steady-state convective diffusion equation

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$$(\mathbf{v}\nabla)c = D\Delta c$$

with boundary conditions expressing the constancy of the concentration far from the lattice and the complete absorption of the material dissolved in the stream at the surfaces of the spheres (D is the diffusion coefficient).

Henceforth we assume that the Peclet number Pe = aU/D is large. This implies that all the main change of concentration will occur in the thin diffusion boundary layer of each sphere in which the tangential transport of material along the surface of the particle can be neglected in comparison with the radial transport, and in the region of diffusion wakes in the neighborhood of the singular streamlines which begin and end on particle surfaces. Therefore, to determine the concentration near a fixed sphere it is necessary to solve the diffusion boundary layer equation with an in-leakage condition which depends on the relative position of the particle in the lattice, and is given by the concentration distribution in the diffusion wake of the upstream sphere [1-3].

From now on we assume that the lattice period satisfies the inequality $b/a \ll \mathrm{Pe}^{1/3}$. Therefore the in-leakage condition for a sphere in the k-th layer is determined by the concentration distribution in the convective boundary layer region of the diffusion wake of the preceding particle in the (k − 1)-th layer.

In the convective boundary layer region the concentration is constant along the streamlines and is determined by the value of the concentration at the exit from the diffusion boundary layer. This permits the reduction of the original problem to the problem of mass transfer of chains of spheres, and using the results of [1-3] and the assumption that the undepleted solution has a concentration c, the following expressions are obtained for the total diffusion fluxes through the particle surfaces:

$$I_k = I_1 [k^{2/3} - (k-1)^{2/3}], \quad I_1 = \frac{(3\pi)^{5/3} A^{1/3}(n)}{2\Gamma(1/3)} a^{4/3} U^{1/3} D^{2/3} c. \tag{1}$$

Taking account of (1) we obtain for the average diffusion flux through the surface of a sphere

$$\langle I \rangle = k^{-1} \sum_{i=1}^{k} I_i = I_1 k^{-1/3}. \tag{2}$$

We assume now that the number of spheres in the lattice is large, i.e., $k \to \infty$, and determine the average concentration distribution along the stream axis. The concentration outside the diffusion boundary layers and wakes will henceforth also be called the concentration in the flow core.

Since the concentration in the flow core varies slowly over distances of the order of a lattice period, a representative volume can be introduced which is substantially smaller than the scale of variation of the concentration, but contains a large number of particles.

We introduce a coordinate x measured along the stream. At the lattice nodes it takes on the values

$$x = x(k) = kn^{-1/3}, \tag{3}$$

where n is the number of spheres per unit volume.

Taking account of (3) and the equation for the concentration in the flow core

$$-U\partial c/\partial x = n\langle I \rangle, \; x = 0, \; c = c_0$$

we obtain from (2) the average concentration distribution along the stream

$$c = c_0 \exp(-Fx^{2/3}), \quad F = 3n^{8/9} A^{1/3}(n) \frac{(3\pi)^{5/3}}{4\Gamma(1/3)} a^{4/3} U^{-2/3} D^{2/3}. \tag{4}$$

We note that Eq. (4) is significantly different from the analogous result for a spatially uniform distribution of spheres [4] in which the concentration depends exponentially on the coordinate along the stream axis. The interaction of diffusion wakes and boundary layers of particles in the lattice causes the average concentration to decrease more slowly, but it will always be larger than the analogous concentration for a random distribution of spheres in a volume.

By using the rather general results in [9] for Stokes flow around an ordered system of identical particles of arbitrary shape when the singular streamlines emerging from the surfaces of particles of each layer fall on particles of the next layer, and using [2], an expression (a generalization of (4)) can be obtained for the concentration in the flow core. The expression for the average concentration distribution in this case differs from Eq. (4) only in the value of the coefficient F.

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FREE LAMINAR CONVECTION OF A LIQUID IN A RIBBED SLOT

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Free convection in the vertical gaps of technical apparatus and constructions often occurs in layers having walls with complex geometry. Transverse projections on one or both walls of a liquid or gas layer may overlap part of the width of the layer. It is obvious that the presence of such projections may change the flow pattern in the gap and may lead to a change in the heat transfer from the hot to the cold wall.

We used the arrangement shown in Fig. 1 to investigate free convection in a vertical channel with projections. The walls were copper plates 1 (plate thickness 15 mm), placed in a container 2 with the working liquid (ethyl alcohol, Pr = 16). In all the experiments we used a channel of height H = 342 mm and depth B = 56 mm. Its width was changed using a thickness-calibrated attachment made of Plexiglas, trapped between the working surfaces of the plates. The temperature of each of the heat exchangers was maintained constant by circulating water from thermostats through the cavity situated behind the working plates. The constancy of the plate temperature along the height was monitored by means of five Nichrome–Constantan thermocouples (diameter 0.2 mm), embedded flush with the working surface.

To measure the temperature in the layer we used a Nichrome-Constantan thermocouple 0.06 mm in diameter. The thermocouple wires in PVC insulation were placed in a thin-walled capillary of stainless steel along the rear vertical end of the layer, which was shifted by means of an external coordinate reference system in a vertical direction with a reading accuracy of 0.1 mm. The junction of the thermocouple was introduced into the middle depth of the layer through the bent end of the capillary. The junction was displaced in a fixed horizontal plane by rotating the metal capillary. The coordinates of the thermocouple were found using a KM-6 cathetometer with an accuracy of 0.03 mm. The emfs of the thermocouples were measured with a R348 low-resistance potentiometer (class 0.002). The thermocouples were calibrated against a standard platinum resistance thermometer with an accuracy of up to 0.01°C in the temperature range 15-60° C.

For hydrodynamic investigations the method of stroboscopic visualization was used. Aluminum powder in the form of spheres with dimensions of 5 μm was used as the marker. Visual observations and photographs of the structure of the flow were made in reflected light through the transparent wall of the container 6. Part of the layer near the middle was illuminated from the side through a narrow 2-mm vertical glass insertion 3 in one of the plates and a transparent window 4 in the side wall of the container. The illuminating flux was

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